# Mathematica 11.3 Integration Test Results

## Test results for the 136 problems in "8.5 Hyperbolic integral functions.m"

### Problem 6: Unable to integrate problem.

$$\int \frac{\text{SinhIntegral}[b\,x]}{x}\,dx$$

Optimal (type 5, 38 leaves, 1 step):

$$\frac{1}{2}\,b\,x\,\text{HypergeometricPFQ}[\{1,1,1\},\{2,2,2\},-b\,x]+\frac{1}{2}\,b\,x\,\text{HypergeometricPFQ}[\{1,1,1\},\{2,2,2\},b\,x]$$

Result (type 8, 10 leaves):

$$\int \frac{\text{SinhIntegral}[b\,x]}{x}\,dx$$

### Problem 39: Unable to integrate problem.

$$\int \frac{\text{Sinh}[b\,x]\,\text{SinhIntegral}[b\,x]}{x^3}\,dx$$

Optimal (type 4, 96 leaves, 14 steps):

$$b^2\,\text{CoshIntegral}[2\,b\,x]-\frac{b\,\text{Cosh}[b\,x]\,\text{Sinh}[b\,x]}{2\,x}-\frac{\text{Sinh}[b\,x]^2}{4\,x^2}-\frac{b\,\text{Sinh}[2\,b\,x]}{4\,x}-\frac{b\,\text{Cosh}[b\,x]\,\text{SinhIntegral}[b\,x]}{2\,x}-\frac{\text{Sinh}[b\,x]\,\text{SinhIntegral}[b\,x]}{2\,x^2}+\frac{1}{4}\,b^2\,\text{SinhIntegral}[b\,x]^2$$

Result (type 8, 14 leaves):

$$\int \frac{\text{Sinh}[b\,x]\,\text{SinhIntegral}[b\,x]}{x^3}\,dx$$

### Problem 47: Unable to integrate problem.

$$\int \frac{\text{Cosh}[b\,x]\,\text{SinhIntegral}[b\,x]}{x^2}\,dx$$

Optimal (type 4, 44 leaves, 7 steps):

$$\text{b CoshIntegral}[2\,b\,x]-\frac{\text{Sinh}[2\,b\,x]}{2\,x}-\frac{\text{Cosh}[b\,x]\,\text{SinhIntegral}[b\,x]}{x}+\frac{1}{2}\,b\,\text{SinhIntegral}[b\,x]^2$$

Result (type 8, 14 leaves):

$$\int\frac{\text{Cosh}[b\,x]\,\text{SinhIntegral}[b\,x]}{x^2}\,dx$$

## Problem 63: Result more than twice size of optimal antiderivative.

$$\int x\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]\,dx$$

Optimal (type 4, 371 leaves, 24 steps):

$$\frac{\text{Cosh}[a-c+(b-d)\,x]}{2\,b\,(b-d)}-\frac{\text{Cosh}[a+c+(b+d)\,x]}{2\,b\,(b+d)}-\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}+$$
$$\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}-\frac{c\,\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d}+$$
$$\frac{c\,\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d}-$$
$$\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}-\frac{\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}+$$
$$\frac{x\,\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]}{b}-\frac{\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]}{b^2}+$$
$$\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}+\frac{\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}$$

Result (type 4, 887 leaves):

$$\frac{1}{4\,b^2\,(b-d)\,d\,(b+d)}\Bigg(2\,b^2\,d\,\text{Cosh}[a-c+b\,x-d\,x]+$$
$$2\,b\,d^2\,\text{Cosh}[a-c+b\,x-d\,x]-2\,b^2\,d\,\text{Cosh}[a+c+(b+d)\,x]+2\,b\,d^2\,\text{Cosh}[a+c+(b+d)\,x]-$$
$$2\,(b^2-d^2)\,\text{CoshIntegral}\left[-\frac{(b-d)\,(c+d\,x)}{d}\right]\left(d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]+b\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\right)+$$
$$2\,(b^2-d^2)\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]\left(d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]+b\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\right)+$$
$$4\,b^3\,d\,x\,\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]-4\,b\,d^3\,x\,\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]-$$
$$4\,b^2\,d\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]+4\,d^3\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]-$$
$$b^3\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b\,c\,d^2\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$

$$b^3 c \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b - d)\,(c + d x)}{d}\right] -$$
$$b^2 d \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b - d)\,(c + d x)}{d}\right] +$$
$$b c d^2 \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b - d)\,(c + d x)}{d}\right] +$$
$$d^3 \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b - d)\,(c + d x)}{d}\right] +$$
$$2 b^3 c \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b + d)\,(c + d x)}{d}\right] -$$
$$2 b c d^2 \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b + d)\,(c + d x)}{d}\right] +$$
$$2 b^2 d \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b + d)\,(c + d x)}{d}\right] -$$
$$2 d^3 \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[\frac{(b + d)\,(c + d x)}{d}\right] +$$
$$b^3 c \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] -$$
$$b^2 d \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] -$$
$$b c d^2 \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] +$$
$$d^3 \,\text{Cosh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] -$$
$$b^3 c \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] +$$
$$b^2 d \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] +$$
$$b c d^2 \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right] -$$
$$d^3 \,\text{Sinh}\left[a - \frac{b c}{d}\right] \text{SinhIntegral}\left[c - \frac{b c}{d} - b x + d x\right]\Bigg)$$

## Problem 66: Result more than twice size of optimal antiderivative.

$$\int x \,\text{Cosh}[a + b x] \,\text{SinhIntegral}[c + d x] \, dx$$

Optimal (type 4, 371 leaves, 24 steps):

$$-\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}+\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}-\frac{\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b^2}+\frac{\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b^2}+\frac{\text{Sinh}\left[a-c+(b-d)\,x\right]}{2\,b\,(b-d)}-\frac{\text{Sinh}\left[a+c+(b+d)\,x\right]}{2\,b\,(b+d)}-\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}-\frac{c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}-\frac{\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]}{b^2}+\frac{x\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]}{b}+\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}+\frac{c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}$$

Result (type 4, 887 leaves) :

$$
\begin{aligned}
&\frac{1}{4\, b^2\, (b-d)\, d\, (b+d)}\\
&\Bigg(-2\,(b^2-d^2)\,\text{CoshIntegral}\Big[-\frac{(b-d)\,(c+d\,x)}{d}\Big]\,\Big(b\,c\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]+d\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\Big)+\\
&\quad 2\,(b^2-d^2)\,\text{CoshIntegral}\Big[\frac{(b+d)\,(c+d\,x)}{d}\Big]\,\Big(b\,c\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]+d\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\Big)+\\
&\quad 2\,b^2\,d\,\text{Sinh}[a-c+b\,x-d\,x]+2\,b\,d^2\,\text{Sinh}[a-c+b\,x-d\,x]-2\,b^2\,d\,\text{Sinh}\big[a+c+(b+d)\,x\big]+\\
&\quad 2\,b\,d^2\,\text{Sinh}\big[a+c+(b+d)\,x\big]-4\,b^2\,d\,\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]+\\
&\quad 4\,d^3\,\text{Cosh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]+4\,b^3\,d\,x\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]-\\
&\quad 4\,b\,d^3\,x\,\text{Sinh}[a+b\,x]\,\text{SinhIntegral}[c+d\,x]-b^3\,c\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]-\\
&\quad b^2\,d\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]+\\
&\quad b\,c\,d^2\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]+d^3\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]-b^3\,c\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]-\\
&\quad b^2\,d\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]+b\,c\,d^2\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]+d^3\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b-d)\,(c+d\,x)}{d}\Big]+\\
&\quad 2\,b^2\,d\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b+d)\,(c+d\,x)}{d}\Big]-2\,d^3\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[\frac{(b+d)\,(c+d\,x)}{d}\Big]+2\,b^3\,c\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b+d)\,(c+d\,x)}{d}\Big]-\\
&\quad 2\,b\,c\,d^2\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[\frac{(b+d)\,(c+d\,x)}{d}\Big]-b^3\,c\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]+b^2\,d\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]+\\
&\quad b\,c\,d^2\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]-d^3\,\text{Cosh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]+b^3\,c\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]-\\
&\quad b^2\,d\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]-b\,c\,d^2\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\\
&\quad\ \text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]+d^3\,\text{Sinh}\Big[a-\frac{b\,c}{d}\Big]\,\text{SinhIntegral}\Big[c-\frac{b\,c}{d}-b\,x+d\,x\Big]\Bigg)
\end{aligned}
$$

## Problem 74: Unable to integrate problem.

$$\int \frac{\text{CoshIntegral}[b\,x]}{x}\,dx$$

Optimal (type 5, 52 leaves, 1 step):

$$-\frac{1}{2}\,b\,x\,\text{HypergeometricPFQ}[\{1,1,1\},\{2,2,2\},-b\,x]+\frac{1}{2}\,b\,x\,\text{HypergeometricPFQ}[\{1,1,1\},\{2,2,2\},b\,x]+\text{EulerGamma}\,\text{Log}[x]+\frac{1}{2}\,\text{Log}[b\,x]^2$$

Result (type 8, 10 leaves):

$$\int\frac{\text{CoshIntegral}[b\,x]}{x}\,dx$$

## Problem 107: Unable to integrate problem.

$$\int\frac{\text{Cosh}[b\,x]\,\text{CoshIntegral}[b\,x]}{x^3}\,dx$$

Optimal (type 4, 96 leaves, 14 steps):

$$-\frac{\text{Cosh}[b\,x]^2}{4\,x^2}-\frac{\text{Cosh}[b\,x]\,\text{CoshIntegral}[b\,x]}{2\,x^2}+\frac{1}{4}\,b^2\,\text{CoshIntegral}[b\,x]^2+b^2\,\text{CoshIntegral}[2\,b\,x]-\frac{b\,\text{Cosh}[b\,x]\,\text{Sinh}[b\,x]}{2\,x}-\frac{b\,\text{CoshIntegral}[b\,x]\,\text{Sinh}[b\,x]}{2\,x}-\frac{b\,\text{Sinh}[2\,b\,x]}{4\,x}$$

Result (type 8, 14 leaves):

$$\int\frac{\text{Cosh}[b\,x]\,\text{CoshIntegral}[b\,x]}{x^3}\,dx$$

## Problem 115: Unable to integrate problem.

$$\int\frac{\text{CoshIntegral}[b\,x]\,\text{Sinh}[b\,x]}{x^2}\,dx$$

Optimal (type 4, 44 leaves, 7 steps):

$$\frac{1}{2}\,b\,\text{CoshIntegral}[b\,x]^2+b\,\text{CoshIntegral}[2\,b\,x]-\frac{\text{CoshIntegral}[b\,x]\,\text{Sinh}[b\,x]}{x}-\frac{\text{Sinh}[2\,b\,x]}{2\,x}$$

Result (type 8, 14 leaves):

$$\int\frac{\text{CoshIntegral}[b\,x]\,\text{Sinh}[b\,x]}{x^2}\,dx$$

## Problem 131: Result more than twice size of optimal antiderivative.

$$\int x\,\text{CoshIntegral}[c+d\,x]\,\text{Sinh}[a+b\,x]\,dx$$

Optimal (type 4, 371 leaves, 24 steps):

$$\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}+$$
$$\frac{x\,\text{Cosh}[a+b\,x]\,\text{CoshIntegral}[c+d\,x]}{b}+\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}+$$
$$\frac{\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b^2}+\frac{\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b^2}-$$
$$\frac{\text{CoshIntegral}[c+d\,x]\,\text{Sinh}[a+b\,x]}{b^2}-\frac{\text{Sinh}\left[a-c+(b-d)\,x\right]}{2\,b\,(b-d)}-\frac{\text{Sinh}\left[a+c+(b+d)\,x\right]}{2\,b\,(b+d)}+$$
$$\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}+\frac{c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}+$$
$$\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}+\frac{c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}$$

Result (type 4, 916 leaves):

$$\frac{1}{4\,b^2\,(b-d)\,d\,(b+d)}\left(2\,b^3\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]-\right.$$
$$2\,b\,c\,d^2\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]+$$
$$2\,b^2\,d\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]-$$
$$2\,d^3\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]+$$
$$2\,(b^2-d^2)\,\text{CoshIntegral}\left[-\frac{(b-d)\,(c+d\,x)}{d}\right]\left(b\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]+d\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\right)+$$
$$4\,d\,(b^2-d^2)\,\text{CoshIntegral}[c+d\,x]\,(b\,x\,\text{Cosh}[a+b\,x]-\text{Sinh}[a+b\,x])-$$
$$2\,b^2\,d\,\text{Sinh}[a-c+b\,x-d\,x]-2\,b\,d^2\,\text{Sinh}[a-c+b\,x-d\,x]-2\,b^2\,d\,\text{Sinh}\left[a+c+(b+d)\,x\right]+$$
$$2\,b\,d^2\,\text{Sinh}\left[a+c+(b+d)\,x\right]+b^3\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$b\,c\,d^2\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^3\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^2\,d\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$b\,c\,d^2\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\,\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$

$$d^3 \text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$2\,b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]-$$
$$2\,d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]+$$
$$2\,b^3\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]-$$
$$2\,b\,c\,d^2\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]+$$
$$b^3\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]-$$
$$b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]-$$
$$b\,c\,d^2\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]+$$
$$d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]-$$
$$b^3\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]+$$
$$b^2\,d\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]+$$
$$b\,c\,d^2\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]-$$
$$\left.d^3\,\text{Sinh}\left[a-\frac{b\,c}{d}\right] \text{SinhIntegral}\left[c-\frac{b\,c}{d}-b\,x+d\,x\right]\right)$$

## Problem 134: Result more than twice size of optimal antiderivative.

$$\int x\,\text{Cosh}[a+b\,x]\,\text{CoshIntegral}[c+d\,x]\,dx$$

Optimal (type 4, 371 leaves, 24 steps):

$$-\frac{\text{Cosh}[a-c+(b-d)\,x]}{2\,b\,(b-d)}-\frac{\text{Cosh}[a+c+(b+d)\,x]}{2\,b\,(b+d)}+$$
$$\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}-\frac{\text{Cosh}[a+b\,x]\,\text{CoshIntegral}[c+d\,x]}{b^2}+$$
$$\frac{\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}+\frac{c\,\text{CoshIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d}+$$
$$\frac{c\,\text{CoshIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]\text{Sinh}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d}+\frac{x\,\text{CoshIntegral}[c+d\,x]\,\text{Sinh}[a+b\,x]}{b}+$$
$$\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b\,d}+\frac{\text{Sinh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{c\,(b-d)}{d}+(b-d)\,x\right]}{2\,b^2}+$$
$$\frac{c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b\,d}+\frac{\text{Sinh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{c\,(b+d)}{d}+(b+d)\,x\right]}{2\,b^2}$$

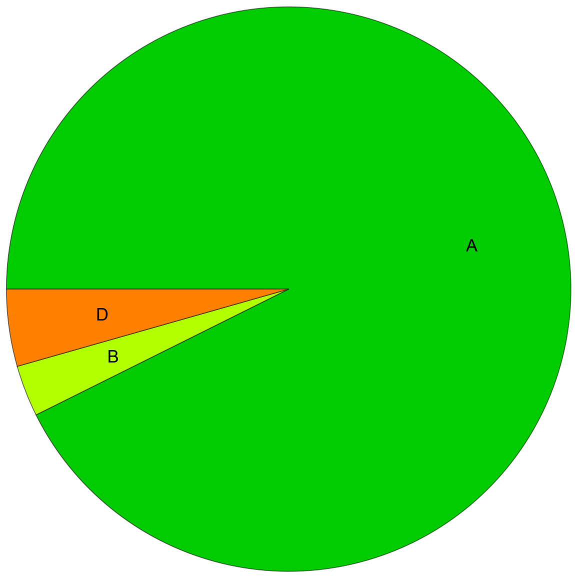
Result (type 4, 916 leaves):

$$\frac{1}{4\,b^2\,(b-d)\,d\,(b+d)}$$
$$\Bigg(-2\,b^2\,d\,\text{Cosh}[a-c+b\,x-d\,x]-2\,b\,d^2\,\text{Cosh}[a-c+b\,x-d\,x]-2\,b^2\,d\,\text{Cosh}[a+c+(b+d)\,x]+$$
$$2\,b\,d^2\,\text{Cosh}[a+c+(b+d)\,x]+2\,b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]-$$
$$2\,d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]+2\,b^3\,c\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]$$
$$\text{Sinh}\left[a-\frac{b\,c}{d}\right]-2\,b\,c\,d^2\,\text{CoshIntegral}\left[\frac{(b+d)\,(c+d\,x)}{d}\right]\text{Sinh}\left[a-\frac{b\,c}{d}\right]+$$
$$2\,(b^2-d^2)\,\text{CoshIntegral}\left[-\frac{(b-d)\,(c+d\,x)}{d}\right]\left(d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]+b\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\right)+$$
$$4\,d\,(b^2-d^2)\,\text{CoshIntegral}[c+d\,x]\,(-\text{Cosh}[a+b\,x]+b\,x\,\text{Sinh}[a+b\,x])+$$
$$b^3\,c\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^2\,d\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$b\,c\,d^2\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$d^3\,\text{Cosh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^3\,c\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]+$$
$$b^2\,d\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$
$$b\,c\,d^2\,\text{Sinh}\left[a-\frac{b\,c}{d}\right]\text{SinhIntegral}\left[\frac{(b-d)\,(c+d\,x)}{d}\right]-$$

$$d^3 \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[\frac{(b-d)\ (c+d x)}{d}\right]+$$
$$2 b^3 c \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[\frac{(b+d)\ (c+d x)}{d}\right]-$$
$$2 b c d^2 \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[\frac{(b+d)\ (c+d x)}{d}\right]+$$
$$2 b^2 d \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[\frac{(b+d)\ (c+d x)}{d}\right]-$$
$$2 d^3 \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[\frac{(b+d)\ (c+d x)}{d}\right]-$$
$$b^3 c \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]+$$
$$b^2 d \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]+$$
$$b c d^2 \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]-$$
$$d^3 \operatorname{Cosh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]+$$
$$b^3 c \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]-$$
$$b^2 d \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]-$$
$$b c d^2 \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]+$$
$$d^3 \operatorname{Sinh}\left[a-\frac{b c}{d}\right] \operatorname{SinhIntegral}\left[c-\frac{b c}{d}-b x+d x\right]\Bigg)$$

# Summary of Integration Test Results

136 integration problems




A - 126 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 0 integration timeouts